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FORTRAN Routines for Spectral Methods

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Introduction

Spectral methods are an efficient tool to recover accurate approximated solutions to ordinary and partial differential equations, in terms of high-degree trigonometric or algebraic polynomials. A lot has been done to study the numerical implementation and the theoretical analysis of convergence of these techniques. A survey of the main results is given for instance in [1], [2], [3], [5], [7].

When building a numerical code using spectral methods, a preliminary part has to be devoted to a series of basic algorithms, not often readily available in the usual program libraries. This requires the user to be a little acquainted with the properties of orthogonal functions. The initialization is however quite standard and the collection of FORTRAN routines presented here is intended to provide the user with a ready software product, in order to allow a smooth start in the developement of a more extensive code.

The *double precision* subroutines are listed in this manual together with a detailed description. Style and notations are those adopted in [4], where *tau* and *collocation* approximations by algebraic polynomials are studied for one-dimensional differential equations. As a matter of fact, this text has been conceived to complement the material contained in [4].

The name of each routine consists of six letters. The third and the fourth denote the kind of polynomial basis considered: JA stands for Jacobi, LE for Legendre, CH for Chebyshev, LA for Laguerre, HE for Hermite. For the routines related with a certain set of collocation nodes, the last two letters denote the type of integration formula originating from these nodes: GA stands for Gauss, GL for Gauss-Lobatto, GR for Gauss-Radau.

In order to fit the mathematical notations, many vectors are dimensioned starting from the component zero. The user is invited to check the vector dimensioning of his main program with the help of the tables provided at the end of the manual.

Some of the routines related to the set of Chebyshev collocation nodes have a second version using the algorithm of the Fast Fourier Transform. In this case, external auxiliary subroutines of the NAG (*Numerical Algorithms Group*) FORTRAN Library are needed.

The routines have been tested when the various parameters fall in the range of standard applications. Of course, for high values of the polynomial degrees, round off errors may occur, and a special care is required in the computations involving Laguerre or Hermite polynomials. The user is suggested to double check the validity of the output results in those situations expected to be critical.

Purposes

GAMMAF: evaluates the real gamma function at a given point

VAJAPO: computes the value and the derivatives of the Jacobi polynomials at a given point

VALEPO: computes the value and the derivatives of the Legendre polynomials at a given point

VACHPO: computes the value and the derivatives of the Chebyshev polynomials at a given point

VALAPO: computes the value and the derivatives of the Laguerre polynomials at a given point

VAHEPO: computes the value and the derivatives of the Hermite polynomials at a given point

VALASF: computes the value and the derivative of the scaled Laguerre functions at a given point

VAHESF: computes the value and the derivative of the scaled Hermite functions at a given point

ZEJAGA: finds the zeroes of the Jacobi polynomials

ZELEGA: finds the zeroes of the Legendre polynomials

ZECHGA: finds the zeroes of the Chebyshev polynomials

ZELAGA: finds the zeroes of the Laguerre polynomials

ZEHEGA: finds the zeroes of the Hermite polynomials

ZEJAGL: finds the nodes of the Jacobi Gauss-Lobatto integration formula

ZELEGL: finds the nodes of the Legendre Gauss-Lobatto integration formula

ZECHGL: finds the nodes of the Chebyshev Gauss-Lobatto integration formula

ZELAGR: finds the nodes of the Laguerre Gauss-Radau integration formula

WEJAGA: finds the weights of the Jacobi Gauss integration formula

WELEGA: finds the weights of the Legendre Gauss integration formula

WECHGA: finds the weights of the Chebyshev Gauss integration formula

WELAGA: finds the weights of the Laguerre Gauss integration formula

WEHEGA: finds the weights of the Hermite Gauss integration formula

WEJAGL: finds the weights of the Jacobi Gauss-Lobatto integration formula

WELEGL: finds the weights of the Legendre Gauss-Lobatto integration formula

WECHGL: finds the weights of the Chebyshev Gauss-Lobatto integration formula

WELAGR: finds the weights of the Laguerre Gauss-Radau integration formula

WECHCC: finds the weights of the Clenshaw-Curtis integration formula

INJAGA: evaluates at a given point the value of a polynomial given at the Jacobi zeroes

INLEGA: evaluates at a given point the value of a polynomial given at the Legendre zeroes

INCHGA: evaluates at a given point the value of a polynomial given at the Chebyshev zeroes

INLAGA: evaluates at a given point the value of a polynomial given at the Laguerre zeroes

INHEGA: evaluates at a given point the value of a polynomial given at the Hermite zeroes

INJAGL: evaluates at a given point the value of a polynomial given at the Jacobi Gauss-Lobatto nodes

INLEGL: evaluates at a given point the value of a polynomial given at the Legendre Gauss-Lobatto nodes

INCHGL: evaluates at a given point the value of a polynomial given at the Chebyshev Gauss-Lobatto nodes

INLAGR: evaluates at a given point the value of a polynomial given at the Laguerre Gauss-Radau nodes

NOLEGA: evaluates the $L^2(-1,1)$ norm and the discrete maximum norm of a polynomial given at the Legendre zeroes

NOCHGA: evaluates the $L^2(-1,1)$ norm (with and without weight function) and the discrete maximum norm of a polynomial given at the Chebyshev zeroes

NOJAGL: evaluates the weighted $L^2(-1,1)$ norm, the quadrature norm, and the discrete maximum norm of a polynomial given at the Jacobi Gauss-Lobatto nodes

NOLEGL: evaluates the $L^2(-1,1)$ norm, the quadrature norm, and the discrete maximum norm of a polynomial given at the Legendre Gauss-Lobatto nodes

NOCHGL: evaluates the $L^2(-1,1)$ norm (with and without weight function), the quadrature norm, and the discrete maximum norm of a polynomial given at the Chebyshev Gauss-Lobatto nodes

COJAGA: evaluates the Jacobi Fourier coefficients of a polynomial given at the Jacobi zeroes

COLEGA: evaluates the Legendre Fourier coefficients of a polynomial given at the Legendre zeroes

COCHGA: evaluates the Chebyshev Fourier coefficients of a polynomial given at the Chebyshev zeroes (without using FFT)

COLAGA: evaluates the Laguerre Fourier coefficients of a polynomial given at the Laguerre zeroes

COHEGA: evaluates the Hermite Fourier coefficients of a polynomial given at the Hermite zeroes

COJAGL: evaluates the Jacobi Fourier coefficients of a polynomial given at the Jacobi Gauss-Lobatto nodes

COLEGL: evaluates the Legendre Fourier coefficients of a polynomial given at the Legendre Gauss-Lobatto nodes

COCHGL: evaluates the Chebyshev Fourier coefficients of a polynomial given at the Chebyshev Gauss-Lobatto nodes (without using FFT)

COLAGR: evaluates the Laguerre Fourier coefficients of a polynomial given at the Laguerre Gauss-Radau nodes

PVJAEX: computes the value and the derivatives at a given point of a polynomial, from its Jacobi Fourier coefficients

PVLEEX: computes the value and the derivatives at a given point of a polynomial, from its Legendre Fourier coefficients

PVCHEX: computes the value and the derivatives at a given point of a polynomial, from its Chebyshev Fourier coefficients

PVLAEX: computes the value and the derivatives at a given point of a polynomial, from its Laguerre Fourier coefficients

PVHEEX: computes the value and the derivatives at a given point of a polynomial, from its Hermite Fourier coefficients

NOJAEX: evaluates the weighted $L^2(-1,1)$ norm of a polynomial, from its Jacobi Fourier coefficients

NOLEEX: evaluates the $L^2(-1,1)$ norm of a polynomial, from its Legendre Fourier coefficients

NOCHEX: evaluates the $L^2(-1,1)$ norm (with and without weight function) of a polynomial, from its Chebyshev Fourier coefficients

NOLAEX: evaluates the weighted $L^2(0,+\infty)$ norm of a polynomial, from its Laguerre Fourier coefficients

NOHEEX: evaluates the weighted $L^2(\mathbf{R})$ norm of a polynomial, from its Hermite Fourier coefficients

COJADE: computes the Jacobi Fourier coefficients of the derivatives of a polynomial, from its Jacobi Fourier coefficients

COLEDE: computes the Legendre Fourier coefficients of the derivatives of a polynomial, from its Legendre Fourier coefficients

COCHDE: computes the Chebyshev Fourier coefficients of the derivatives of a polynomial, from its Chebyshev Fourier coefficients

COLADE: computes the Laguerre Fourier coefficients of the derivatives of a polynomial, from its Laguerre Fourier coefficients

COHEDE: computes the Hermite Fourier coefficients of the derivatives of a polynomial, from its Hermite Fourier coefficients

DEJAGA: computes the derivative at the Jacobi zeroes of a polynomial given at the Jacobi zeroes

DELAGA: computes the derivative at the Laguerre zeroes of a polynomial given at the Laguerre zeroes

DEHEGA: computes the derivative at the Hermite zeroes of a polynomial given at the Hermite zeroes

DEJAGL: computes the derivative at the Jacobi Gauss-Lobatto nodes of a polynomial given at the Jacobi Gauss-Lobatto nodes

DELEGL: computes the derivative at the Legendre Gauss-Lobatto nodes of a polynomial given at the Legendre Gauss-Lobatto nodes

DECHGL: computes the derivative at the Chebyshev Gauss-Lobatto nodes of a polynomial given at the Chebyshev Gauss-Lobatto nodes (without using FFT)

DELAGR: computes the derivative at the Laguerre Gauss-Radau nodes of a polynomial given at the Laguerre Gauss-Radau nodes

DMJAGL: gives the entries of the derivative matrix relative to the Jacobi Gauss-Lobatto nodes

DMLEGL: gives the entries of the derivative matrix relative to the Legendre Gauss-Lobatto nodes

DMCHGL: gives the entries of the derivative matrix relative to the Chebyshev Gauss-Lobatto nodes

DMLAGR: gives the entries of the derivative matrix relative to the Laguerre Gauss-Radau nodes

FCCHGA: evaluates the Chebyshev Fourier coefficients of a polynomial given at the Chebyshev zeroes (using FFT).

FCCHGL: evaluates the Chebyshev Fourier coefficients of a polynomial given at the Chebyshev Gauss-Lobatto nodes (using FFT).

FVCHGL: computes the values at the Chebyshev Gauss-Lobatto nodes of a polynomial, from its Chebyshev Fourier coefficients (using FFT).

FDCHGL: computes the derivative at the Chebyshev Gauss-Lobatto nodes of a polynomial given at the Chebyshev Gauss-Lobatto nodes (using FFT).

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